Quadratic Tilt-Excess Decay and Strong Maximum Principle for Varifolds

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Abstract. In this paper, we prove that integral \( n \)-varifolds \( \mu \) in codimension 1 with \( H_\mu \in L^p_\text{loc}(\mu), \, p > n, \, p \geq 2 \) have quadratic tilt-excess decay

\[
tillex_\mu(x, \varrho, T_x \mu) = O_\varrho(\varrho^2)
\]

for \( \mu \)-almost all \( x \), and a strong maximum principle which states that these varifolds cannot be touched by smooth manifolds whose mean curvature is given by the weak mean curvature \( H_\mu \), unless the smooth manifold is locally contained in the support of \( \mu \).

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1. – Introduction

The tilt-excess and height-excess of a rectifiable \( n \)-varifold \( \mu \) measures the local deviation of the tangent plane to a given plane

\[
tillex_\mu(x, \varrho, T) := \varrho^{-n} \int_{B_\varrho(x)} \| T_\xi \mu - T \|^2 \, d\mu(\xi)
\]

and the distance of the support to a given plane

\[
heightex_\mu(x, \varrho, T) := \varrho^{-n-2} \int_{B_\varrho(x)} \text{dist}(\xi - x, T)^2 d\mu(\xi),
\]

respectively. For notions in geometric measure theory, we refer to [F] and [Sim].

Tilt-excess decay estimates for rectifiable varifolds were established by Allard in [All72, Theorem 8.16] for the proof of his Regularity Theorem for unit-density.