Varieties with $P_3 (X) = 4$ and $q(X) = \dim(X)$

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Abstract. We classify varieties with $P_3 (X) = 4$ and $q(X) = \dim(X)$.

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1. – Introduction

Let $X$ be a smooth complex projective variety. When $\dim(X) \geq 3$ it is very hard to classify such varieties in terms of their birational invariants. Surprisingly, when $X$ has many holomorphic 1-forms, it is sometimes possible to achieve classification results in any dimension. In [Ka], Kawamata showed that: If $X$ is a smooth complex projective variety with $\kappa(X) = 0$ then the Albanese morphism $\alpha : X \rightarrow A(X)$ is surjective. If moreover, $q(X) = \dim(X)$, then $X$ is birational to an abelian variety. Subsequently, Kollár proved an effective version of this result (cf. [Ko2]): If $X$ is a smooth complex projective variety with $P_m(X) = 1$ for some $m \geq 4$, then the Albanese morphism $\alpha : X \rightarrow A(X)$ is surjective. If moreover, $q(X) = \dim(X)$, then $X$ is birational to an abelian variety. These results where further refined and expanded as follows:

**Theorem 1.1** (cf. [CH1], [CH3], [HP], [Hac2]). If $P_m(X) = 1$ for some $m \geq 2$ or if $P_3(X) \leq 3$, then the Albanese morphism $\alpha : X \rightarrow A(X)$ is surjective. If moreover $q(X) = \dim(X)$, then:

1. If $P_m(X) = 1$ for some $m \geq 2$, then $X$ is birational to an abelian variety.
2. If $P_3(X) = 2$, then $\kappa(X) = 1$ and $X$ is birational to a double cover of its Albanese variety.
3. If $P_3(X) = 3$, then $\kappa(X) = 1$ and $X$ is birational to a bi-double cover of its Albanese variety.

In this paper we will prove a similar result for varieties with $P_3(X) = 4$ and $q(X) = \dim(X)$. We start by considering the following examples:

**Example 1.** Let $G$ be a group acting faithfully on a curve $C$ and acting faithfully by translations on an abelian variety $\tilde{K}$, so that $C / G = E$ is an