Summability of semicontinuous supersolutions to a quasilinear parabolic equation

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Abstract. We study the so-called $p$-superparabolic functions, which are defined as lower semicontinuous supersolutions of a quasilinear parabolic equation. In the linear case, when $p = 2$, we have supercaloric functions and the heat equation. We show that the $p$-superparabolic functions have a spatial Sobolev gradient and a sharp summability exponent is given.

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1. Introduction

The objective of our work is a class of unbounded “supersolutions” of the partial differential equation

$$\frac{\partial u}{\partial t} = \text{div}(|\nabla u|^{p-2}\nabla u), \quad 1 < p < \infty. \quad (1.1)$$

The functions that we have in mind are pointwise defined as lower semicontinuous functions obeying the comparison principle with respect to the solutions of (1.1). They are called $p$-superparabolic functions. In the linear case $p = 2$ we have the ordinary heat equation and supercaloric functions. In the stationary case supercaloric functions are nothing else but superharmonic functions, well-known in the classical potential theory. The $p$-superparabolic functions play an important role in the Perron method in a nonlinear potential theory, described in [7]. We seize the opportunity to mention that the $p$-superparabolic functions are precisely the viscosity supersolutions of (1.1), which fact will not be considered in the present work, see [5].

It is important to observe that in their definition (to be given below) the $p$-superparabolic functions are not required to have any derivatives. The only tie

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