Doubling constant mean curvature tori in $S^3$

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Abstract. The Clifford tori in $S^3$ constitute a one-parameter family of flat, two-dimensional, constant mean curvature (CMC) submanifolds. This paper demonstrates that new, topologically non-trivial CMC surfaces resembling a pair of neighbouring Clifford tori connected at a sub-lattice consisting of at least two points by small catenoidal bridges can be constructed by perturbative PDE methods. That is, one can create a submanifold that has almost everywhere constant mean curvature by gluing a re-scaled catenoid into the neighbourhood of each point of a sub-lattice of the Clifford torus; and then one can show that a constant mean curvature perturbation of this submanifold does exist.

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1. Introduction and statement of results

CMC surfaces. A constant mean curvature (CMC) surface $\Sigma$ contained in an ambient Riemannian manifold $X$ has the property that its mean curvature with respect to the induced metric is constant. This property ensures that the surface area of $\Sigma$ is a critical value of the area functional for surfaces of $X$ subject to an enclosed-volume constraint. Constant mean curvature surfaces have been objects of great interest since the beginnings of modern differential geometry. Classical examples of non-trivial CMC surfaces in $\mathbb{R}^3$ are the sphere, the cylinder and the Delaunay surfaces, and for a long while these were the only known CMC surfaces. In fact, a result of Alexandrov [2] states that the only compact, connected, embedded CMC surfaces in $\mathbb{R}^3$ are spheres.

In recent decades, the theory of CMC surfaces in $\mathbb{R}^3$ has progressed considerably. In 1986, Wente discovered a family of compact, immersed CMC tori [20]; these have been thoroughly studied also in [17]. Since then, several parallel sequences of ideas have led to a profusion of new CMC surfaces. First, the techniques used by Wente have culminated in a representation for CMC surfaces in $\mathbb{R}^3$ akin to the classically-known Weierstraß representation of minimal surfaces in which a harmonic but not anti-conformal map from a Riemann surface to the unit sphere becomes the Gauß map of a CMC immersion into $\mathbb{R}^3$ from which the immersion

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