Abstract. The most accurate determinateness criteria for the multivariate moment problem require the density of polynomials in a weighted Lebesgue space of a generic representing measure. We propose a relaxation of such a criterion to the approximation of a single function, and based on this condition we analyze the impact of the geometry of the support on the uniqueness of the representing measure. In particular we show that a multivariate moment sequence is determinate if its support has dimension one and is virtually compact; a generalization to higher dimensions is also given. Among the one-dimensional sets which are not virtually compact, we show that at least a large subclass supports indeterminate moment sequences. Moreover, we prove that the determinateness of a moment sequence is implied by the same condition (in general easier to verify) of the push-forward sequence via finite morphisms.

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1. Introduction

Let \( \mu \) be a positive measure on the real line, rapidly decreasing at infinity. The asymptotic expansion of the associated Markov function

\[
\int_{\mathbb{R}} \frac{d\mu(t)}{t-z} \approx -\frac{a_0}{z} - \frac{a_1}{z^2} - \cdots, \quad \text{Im}(z) > 0,
\]

and the uniquely determined continued fraction development

\[
-\frac{a_0}{z} - \frac{a_1}{z^2} - \cdots = -\frac{a_0}{z - \alpha_0 - \frac{\beta_0^2}{z - \alpha_1 - \frac{\beta_1^2}{z - \alpha_2 - \frac{\beta_2^2}{\cdots}}}, \quad \alpha_k, \beta_k \in \mathbb{R},
\]

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