**Counting lines on surfaces**

**SAMUEL BOISSIÈRE AND ALESSANDRA SARTI**

**Abstract.** This paper deals with surfaces with many lines. It is well-known that a cubic contains 27 of them and that the maximal number for a quartic is 64. In higher degree the question remains open. Here we study classical and new constructions of surfaces with high number of lines. We obtain a symmetric octic with 352 lines, and give examples of surfaces of degree $d$ containing a sequence of $d(d-2)+4$ skew lines.

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![Cubic surface with 27 lines](http://enriques.mathematik.uni-mainz.de/surf/logo.jpg)

1. **Introduction**

Motivation for this paper is the article of 1943 by Segre [12] which studies the following classical problem: What is the maximum number of lines that a surface of degree $d$ in $\mathbb{P}^3$ can have? Segre answers this question for $d = 4$ by using some nice geometry, showing that it is exactly 64. For the degree three it is a classical result that each smooth cubic in $\mathbb{P}^3$ contains 27 lines, but for $d \geq 5$ this number is still not known. In this case, Segre shows in [12] that the maximal number is less

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