Degenerate elliptic equations with nonlinear boundary conditions and measures data

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Dedicated to our friend Lucio Boccardo on the occasion of his 60th birthday.

Abstract. In this paper we study the questions of existence and uniqueness of solutions for equations of type $-\text{div} \, a(x, Du) + \gamma(u) \ni \mu_1$, posed in an open bounded subset $\Omega$ of $\mathbb{R}^N$, with nonlinear boundary conditions of the form $a(x, Du) \cdot \eta + \beta(u) \ni \mu_2$. The nonlinear elliptic operator $\text{div} \, a(x, Du)$ is modeled on the $p$-Laplacian operator $\Delta_p(u) = \text{div} \, (|Du|^{p-2}Du)$, with $p > 1$, $\gamma$ and $\beta$ are maximal monotone graphs in $\mathbb{R}^2$ such that $0 \in \gamma(0) \cap \beta(0)$ and the data $\mu_1$ and $\mu_2$ are measures.

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1. Introduction

The purpose of this paper is to establish existence and uniqueness of solutions for a degenerate elliptic problem with nonlinear boundary condition of the form

\[
(S_{\mu_1, \mu_2}^{\gamma, \beta}) \begin{cases} 
-\text{div} \, a(x, Du) + \gamma(u) \ni \mu_1 & \text{in } \Omega \\
 a(x, Du) \cdot \eta + \beta(u) \ni \mu_2 & \text{on } \partial \Omega,
\end{cases}
\]

where $\Omega$ is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial \Omega$, the function $a : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function with growth of order $p - 1$ ($p > 1$) with respect to the gradient, satisfying the classical Leray-Lions conditions, $\eta$ is the unit outward normal on $\partial \Omega$ and $\mu_1, \mu_2$ are measures such that $\mu_1 = \mu_1 \mathbb{1}_\Omega$, $\mu_2 = \mu_2 \mathbb{1}_{\partial \Omega}$ and $\mu_1 + \mu_2$ is a diffuse measure (it does not charge sets of zero $p$-capacity). The nonlinearities $\gamma$ and $\beta$ are maximal monotone

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