Periodic solutions of forced Kirchhoff equations

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Abstract. We consider the Kirchhoff equation for a vibrating body, in any dimension, in the presence of a time-periodic external forcing with period $2\pi/\omega$ and amplitude $\epsilon$. We treat both Dirichlet and space-periodic boundary conditions, and both analytic and Sobolev regularity.

We prove the existence, regularity and local uniqueness of time-periodic solutions, using a Nash-Moser iteration scheme. The results hold for parameters $(\omega, \epsilon)$ in a Cantor set with asymptotically full measure as $\epsilon \to 0$.

Mathematics Subject Classification (2000): 35L70 (primary); 45K05, 35B10, 37K55 (secondary).

1. Introduction

We consider the Kirchhoff equation

$$u_{tt} - \Delta u \left(1 + \int_{\Omega} |\nabla u|^2 \, dx\right) = \epsilon g(x, t), \quad x \in \Omega, \ t \in \mathbb{R}, \quad (1.1)$$

where $g$ is a time-periodic external forcing with period $2\pi/\omega$, $\epsilon$ is an amplitude parameter, and the displacement $u : \Omega \times \mathbb{R} \to \mathbb{R}$ is the unknown.

We consider both Dirichlet boundary conditions

$$u(x, t) = 0 \quad \forall \ x \in \partial \Omega, \ t \in \mathbb{R}, \quad (1.2)$$

where $\Omega \subset \mathbb{R}^d$, $d \geq 1$, is a bounded, connected open set with smooth boundary, and periodic boundary conditions

$$u(x, t) = u(x + 2\pi m, t) \quad \forall \ m \in \mathbb{Z}^d, \ x \in \mathbb{R}^d, \ t \in \mathbb{R}, \quad (1.3)$$

where $\Omega = (0, 2\pi)^d$.

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