Classical solutions and stability results
for Stokesian Hele-Shaw flows

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Abstract. In this paper we study a mathematical model for the motion of a Stokesian fluid in a Hele-Shaw cell surrounded by a gas at uniform pressure. The model is based on a non-Newtonian version of Darcy’s law for the bulk fluid, as suggested in [9, 12].

Besides a general existence and uniqueness result for classical solutions, it is also shown that classical solutions exist globally and tend to circles exponentially fast, provided the initial data is sufficiently close to a circle. Finally, our analysis discloses the influence of surface tension and the effective viscosity on the rate of convergence.

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1. Introduction and main results

Despite their importance in applications, the mathematical understanding of moving boundary problems for non-Newtonian fluids is far from being complete. In this paper we offer an analytic framework in which two-dimensional Stokesian fluids\(^1\) with a free interface may be studied. We investigate the dynamic behaviour of such a fluid located between two parallel and transparent plates in a horizontal Hele-Shaw cell. Relative to some typical lateral dimension on the plate, the distance between these plates is assumed to be small so that we shall consider planar flows in the following. This setting is widely used both in experimental and theoretical work, cf. [9] and the references therein.

The motivation of our research is twofold. One the one hand we provide an analytic framework which guarantees well-posedness of the full flow problem for general data. This result may serve as the theoretical justification of numerical studies or formal expansions. On the other hand we give some insight in the dynamic behaviour of the flow near equilibria. As a main result we show that circles are exponential stable under small perturbations. However, in contrast to the periodic

\(^1\) In a Stokesian fluid the stress tensor is a continuous function of the deformation. The Newtonian fluid is a linear Stokesian fluid. Particularly, the viscosity \(\mu\) is constant in this case.

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