L-series and Hurwitz zeta functions associated with the universal formal group

PIERGIULIO TEMPESTA

Abstract. The properties of the universal Bernoulli polynomials are illustrated and a new class of related L-functions is constructed. A generalization of the Riemann-Hurwitz zeta function is also proposed.

Mathematics Subject Classification (2010): 11M41 (primary); 55N22 (secondary).

1. Introduction

The aim of this article is to establish a connection between the theory of formal groups on one side and a class of generalized Bernoulli polynomials and Dirichlet series on the other side. Some of the results of this paper were announced in the communication [26].

We will prove that the correspondence between the Bernoulli polynomials and the Riemann zeta function can be extended to a larger class of polynomials, by introducing the universal Bernoulli polynomials and the associated Dirichlet series. Also, in the same spirit, generalized Hurwitz zeta functions are defined.

Let $R$ be a commutative ring with identity, and $R \{x_1, x_2, \ldots \}$ be the ring of formal power series in $x_1, x_2, \ldots$ with coefficients in $R$. We recall that a commutative one-dimensional formal group law over $R$ is a formal power series $\Phi(x, y) \in R \{x, y\}$ such that

1) $\Phi(x, 0) = \Phi(0, x) = x$

2) $\Phi(\Phi(x, y), z) = \Phi(x, \Phi(y, z))$.

When $\Phi(x, y) = \Phi(y, x)$, the formal group law is said to be commutative. The existence of an inverse formal series $\varphi(x) \in R \{x\}$ such that $\Phi(x, \varphi(x)) = 0$ follows from the previous definition.

As is well known, formal groups are relevant in many branches of mathematics, especially in the theory of elliptic curves [25], in algebraic topology [5, 21], in analytic number theory [15] and in combinatorics [3].

This research has been supported by the Ministerio de Ciencia e Innovación, Spain, research project FIS2008–00200.

Received June 30, 2008; accepted in revised form March 22, 2009.