Homogenization of processes in nonlinear visco-elastic composites

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Abstract. The constitutive behaviour of a multiaxial visco-elastic material is here represented by the nonlinear relation

$$
\varepsilon - A(x) : \int_0^t \sigma(x, \tau) \, d\tau \in \alpha(\sigma, x),
$$

which generalizes the classical Maxwell model of visco-elasticity of fluid type. Here $\alpha(\cdot, x)$ is a (possibly multivalued) maximal monotone mapping, $\sigma$ is the stress tensor, $\varepsilon$ is the linearized strain tensor, and $A(x)$ is a positive-definite fourth-order tensor. The above inclusion is here coupled with the quasi-static force-balance law, $-\text{div} \, \sigma = \bar{f}$. Existence and uniqueness of the weak solution are proved for a boundary-value problem.

Convergence to a two-scale problem is then derived for a composite material, in which the functions $\alpha$ and $A$ periodically oscillate in space on a short length-scale. It is proved that the coarse-scale averages of stress and strain solve a single-scale homogenized problem, and that conversely any solution of this problem can be represented in that way. The homogenized constitutive relation is represented by the minimization of a time-integrated functional, and is rather different from the above constitutive law. These results are also retrieved via De Giorgi’s notion of $\Gamma$-convergence. These conclusions are at variance with the outcome of so-called analogue models, that rest on an (apparently unjustified) mean-field-type hypothesis.

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1. Introduction

In this work we deal with processes in nonlinear visco-elastic composite materials of fluid type, and illustrate a method of homogenization based on the use of two length-scales.

A nonlinear visco-elastic law. Throughout this paper we make the assumption of infinitesimal displacements, and use the linearized strain-tensor $\varepsilon$. Denoting

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