Least energy nodal solution of a singular perturbed problem with jumping nonlinearity

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Abstract. In this paper we study the asymptotic behavior of the least energy nodal solution of a problem with a jumping nonlinearity.

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1. Introduction

There has been a considerable interest to understand the asymptotic behavior of positive solutions of the elliptic problem

$$
\begin{cases}
\varepsilon^2 \Delta u - u + f(u) = 0 & \text{in } \Omega \\
u = 0 & \text{on } \partial \Omega 
\end{cases}
$$

(1.1)

where $\varepsilon > 0$ is a parameter, $f$ is a superlinear function, $\Omega$ is a smooth bounded domain in $\mathbb{R}^N$. Let $F(u) = \int_0^u f(t) dt$. In this paper, we consider the problem

$$
\begin{cases}
\varepsilon^2 \Delta u - \lambda_1 u^+ + \lambda_2 u^- + f(u) = 0 & \text{in } \Omega \\
u^\pm \neq 0 & \text{in } \Omega \\
u = 0 & \text{on } \partial \Omega 
\end{cases}
$$

(1.2)

where $\lambda_1 > 0, \lambda_2 > 0$ with $\lambda_1 \neq \lambda_2$, and $u^\pm = \max\{\pm u, 0\}$. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function satisfying:

(f1) $f(t) = o(t)$ as $t \to 0$;

(f2) $f(t) = O(|t|^p)$ as $t \to +\infty$ for some $p \in \left(1, \frac{N+2}{N-2}\right)$ if $N \geq 3$ and $p > 1$ if $N = 1, 2$;

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