Cauchy problem and quasi-stationary limit
for the Maxwell-Landau-Lifschitz
and Maxwell-Bloch equations

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Abstract. In this paper we continue the investigation of the Maxwell-Landau-
Lifschitz and Maxwell-Bloch equations. In particular we extend some previous
results about the Cauchy problem and the quasi-stationary limit to the case where
the magnetic permeability and the electric permittivity are variable.

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1. Introduction

The models. This paper deals with two physical models which describe the prop-
agation of electromagnetic waves, that is of the magnetic field $H$ and of the electric
field $E$, in some special medium which occupies an open subset $\Omega$ of $\mathbb{R}^3$, with mag-
netic permeability $\mu$ and electric permittivity $\varepsilon$. In both cases we denote by $\overline{f}$ the
extension of a function $f$ by 0 outside the set $\Omega$. The time variable is $t \geq 0$, and
the space variable is $x \in \mathbb{R}^3$.

The first model refers to Maxwell-Landau-Lifschitz equations (see [10] and
[28] for Physics references). The magnetic field $H$ and the electric field $E$ satisfy
the Maxwell equations in $\mathbb{R}^3$:

\[
\begin{align*}
\mu \partial_t H + \text{curl } E &= -\mu \partial_t \overline{M}, \\
\varepsilon \partial_t E - \text{curl } H &= 0, \\
\text{div } \mu (H + \overline{M}) &= 0, \\
\text{div } \varepsilon E &= 0,
\end{align*}
\]

(1.1)

where $M$ stands for the magnetic moment in the ferromagnet $\Omega$ and takes values in
the unit sphere of $\mathbb{R}^3$. It is solution to the Landau-Lifschitz equation:

\[
\partial_t M = \gamma M \wedge H_T - \alpha M \wedge (M \wedge H_T) \quad \text{for } x \in \Omega,
\]

(1.2)

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