Convergence in capacity on compact Kähler manifolds

SŁAWOMIR DINEW AND PHẠM HOÀNG HIỆP

Abstract. The aim of this note is to study the convergence in capacity for functions in the class $\mathcal{E}(X, \omega)$. We study the problem under several restrictions on the Monge-Ampère measures of the functions considered, such as common domination by a fixed measure or control on the variation.

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1. Introduction

In [2, 3] Bedford and Taylor laid down the foundations of the theory of the complex Monge-Ampère operator which is nowadays a central part of pluripotential theory. In [3] the notion of relative capacity $C_n$ was introduced (see Section 2 for the definitions of all the notions appearing in this note). Initially Bedford and Taylor used this capacity to solve deep problems concerning small sets in pluripotential theory. It was soon realized, however, that capacities are very useful technical tools in solving Monge-Ampère equations with singular data. Especially the discovery of Xing [21], who proved that the complex Monge-Ampère operator is continuous with respect to convergence in capacity, attracted much interest. This is in contrast to convergence in $L^p$, $1 < p < \infty$, since it is known [8] that the Monge-Ampère operator is discontinuous with respect to such topology. Recently convergence in capacity in the setting of domains in $\mathbb{C}^n$ (which for brevity will be referred to as the local setting in this note) was studied by many authors. We refer to [4–6, 9, 15, 17], which is by far an incomplete list of recent contributions, where the reader may obtain a complete picture of the developments in the field.

Quite recently alternative pluripotential theory on compact Kähler manifolds was developed by Guedj and Zeriahi [12, 13] and Kołodziej [18]. It should be noted

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