Lipschitz surfaces, perimeter and trace theorems for BV functions in Carnot-Carathéodory spaces

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Abstract. We introduce intrinsic Lipschitz hypersurfaces in Carnot-Carathéodory spaces and prove that intrinsic Lipschitz domains have locally finite perimeter. We also show the existence of a boundary trace operator for functions with bounded variation on Lipschitz domains and obtain extension results for such functions. In particular, we characterize their trace space.

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1. Introduction and statement of the main results

In the last few years there has been an increasing interest towards analysis and geometry in metric spaces and, in particular, towards geometric measure theory and the study of spaces like those of Sobolev or bounded variation (BV) functions. In this paper we would like to give a contribution in these two directions, by dealing with the study of “Lipschitz regular” hypersurfaces and their relationship with the perimeter measure, and by establishing trace and extension theorems for BV functions in a metric setting. Our framework will be that of a Carnot-Carathéodory (CC) space, i.e., the space \( \mathbb{R}^n \) endowed with the CC distance \( d \) arising from a family \( X = (X_1, \ldots, X_m) \) of smooth vector fields. See Section 2 for precise definitions.

In the setting of Carnot groups (see Section 3 for the definition), intrinsic Lipschitz surfaces have been introduced in [36,38] as graphs of intrinsic Lipschitz maps between complementary subgroups. For the case of codimension one, we propose here a new definition of Lipschitz surface which agrees with the previous one in Carnot groups (see Theorem 3.2) and can be stated in the more general framework of CC spaces.

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