Quantitative uniqueness estimates for the shallow shell system
and their application to an inverse problem

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Abstract. In this paper we derive some quantitative uniqueness estimates for the shallow shell equations. Our proof relies on appropriate Carleman estimates. For applications, we consider the size estimate inverse problem.

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1. Introduction

In this work we study a quantitative uniqueness for the shallow shell system and its application to the inverse problem of estimating the size of an embedded inclusion by boundary measurements. To begin, we let $\Omega$ be a bounded domain in $\mathbb{R}^2$. Without loss of generality, we assume $0 \in \Omega$. Let $\hat{\theta} : \overline{\Omega} \rightarrow \mathbb{R}$ satisfy an appropriate regularity assumption which will be specified later. For a shallow shell, its middle surface is described by $\{(x_1, x_2, \varepsilon \rho_0 \hat{\theta}(x_1, x_2)) : \chi_1, x_2 \in \overline{\Omega}\}$ for $\varepsilon > 0$, where $\rho_0 > 0$ is the characteristic length of $\Omega$ (see Section 3.1). From now on, we set $\theta = \rho_0 \hat{\theta}$. Let $u = (u_1, u_2, u_3) = (u', u_3) : \Omega \rightarrow \mathbb{R}^3$ represent the displacement vector of the middle surface. Then $u$ satisfies the following equations:

$$
\begin{align*}
-\partial_j n_{ij}^\theta(u) & = 0 \quad \text{in} \quad \Omega, \\
\partial_i^2 m_{ij}(u_3) - \partial_j (n_{ij}^\theta(u) \partial_i \theta) & = 0 \quad \text{in} \quad \Omega,
\end{align*}
$$

where

$$
\begin{align*}
m_{ij}(u_3) &= \rho_0^2 \left\{ \frac{4\lambda \mu}{3(\lambda + 2\mu)} (\Delta u_3) \delta_{ij} + \frac{4\mu}{3} \partial_i^2 u_3 \right\}, \\
n_{ij}^\theta(u) &= \frac{4\lambda \mu}{\lambda + 2\mu} \varepsilon_{kk}^\theta(u) \delta_{ij} + 4\mu \varepsilon_{ij}^\theta(u), \\
\varepsilon_{ij}^\theta(u) &= \frac{1}{2} (\partial_i u_j + \partial_j u_i + (\partial_i \theta) \partial_j u_3 + (\partial_j \theta) \partial_i u_3),
\end{align*}
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