Unprojection and deformations
of tertiary Burniat surfaces

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Abstract. We construct a 4-dimensional family of surfaces of general type with
$p_g = 0$ and $K^2 = 3$ and fundamental group $\mathbb{Z}/2 \times Q_8$, where $Q_8$ is the quater-
nion group. The family constructed contains the Burniat surfaces with $K^2 = 3$.
Additionally, we construct the universal coverings of the surfaces in our family
as complete intersections on $(\mathbb{P}^1)^4$ and we also give an action of $\mathbb{Z}/2 \times Q_8$ on
$(\mathbb{P}^1)^4$ lifting the natural action on the surfaces.

The strategy is the following. We consider an étale $(\mathbb{Z}/2)^3$-cover $T$ of a
surface with $p_g = 0$ and $K^2 = 3$ and assume that it may be embedded in a Fano
3-fold $V$. We construct $V$ by using the theory of parallel unprojection. Since $V$ is
an Enriques–Fano 3-fold, considering its Fano cover yields the simple description
of the above universal covers.

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1. Introduction

A Burniat surface is the minimal resolution of singularities of a bidouble cover,
\textit{i.e.,} a finite flat Galois morphism with Galois group $(\mathbb{Z}/2)^2$, of the projective plane
branched along the divisors

$$D_1 = A_1 + A_2 + A_3, \quad D_2 = B_1 + B_2 + B_3, \quad D_3 = C_1 + C_2 + C_3,$$

where $A_1, B_1, C_1$ form a triangle with vertices $x_1, x_2, x_3$, $A_1, A_2, A_3$ are lines
through $x_1$, $B_1, B_2, B_3$ are lines through $x_2$ and $C_1, C_2, C_3$ are lines through $x_3$
(see Figure 1.1). Burniat surfaces were first constructed by Burniat [13], though a
substantial part of the initial study of these surfaces was done about 10 years later by
Peters [29]. They have an equivalent description known as the Inoue surfaces [17],
given as the quotient of a divisor in the product of three elliptic curves by a finite
group. See [3] for an excellent introduction to the subject of Burniat surfaces.

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