Dominated chain recurrent class with singularities

CHRISTIAN BONATTI, SHAOBO GAN AND DAWEI YANG

Abstract. We prove that for $C^1$ generic three-dimensional vector fields, dominated chain recurrent classes with singularities and periodic orbits are singular hyperbolic.

Mathematics Subject Classification (2010): 37C10 (primary); 37D30 (secondary).

1. Introduction

Hyperbolicity has been introduced by Smale [24] for understanding chaotic dynamical behavior and it remains a very important concept for understanding even non-hyperbolic systems. For flows on 3-manifolds, the Lorenz attractor has been the first discovered robustly non hyperbolic system, first announced using numerical experiment [13], and then rigorously modeled in [1, 9, 10] by the so-called geometric model of Lorenz attractor. Lorenz-like attractors share many properties with hyperbolic attractors but involve singular and non-singular orbits in a same transitive dynamics.

In [17] Morales, Pacifico and Pujals introduced singular hyperbolicity to formalize the hyperbolic properties of Lorenz attractors and their generalizations, including the singularities they contain.

The aim of this paper is to give a $C^1$-generic characterization of the singular hyperbolicity by a weaker notion called dominated splitting. In order to state precisely our results, we need some definitions.

1.1. Definitions: singular hyperbolicity, partial hyperbolicity and dominated splittings

Given a closed Riemannian manifold $M$, we denote by $\mathcal{X}^r (M)$ the space of the $C^r$ vector fields on $M$ with the usual $C^r$ norm.

D. Yang thanks the support of NSFC 11001101 and Ministry of Education of P. R. China 20100061120098 and 200903279. S. Gan is supported by 973 program 2011CB808002 and NSFC 11025101. Received June 18, 2012; accepted November 12, 2012.