An existence theorem for steady Navier-Stokes equations in the axially symmetric case

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Abstract. We study the nonhomogeneous boundary value problem for the Navier-Stokes equations of steady motion of a viscous incompressible fluid in a bounded three-dimensional domain with multiply connected boundary. We prove that this problem has a solution in some axially symmetric cases, in particular, when all components of the boundary intersect the axis of symmetry.

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1. Introduction

Let $\Omega$ be a bounded domain in $\mathbb{R}^3$ with Lipschitz boundary $\partial \Omega = \Gamma_0 \cup \ldots \cup \Gamma_N$, consisting of $N + 1$ disjoint connected components $\Gamma_j$. Consider the stationary Navier–Stokes system with nonhomogeneous boundary conditions

$$\begin{align*}
-\nu \Delta u + (u \cdot \nabla) u + \nabla p &= 0 \quad \text{in } \Omega, \\
\text{div } u &= 0 \quad \text{in } \Omega, \\
u \ &= \ a \quad \text{on } \partial \Omega.
\end{align*}$$

The continuity equation (1.12) implies the compatibility condition

$$\int_{\partial \Omega} a \cdot n \, dS = \sum_{j=0}^{N} \int_{\Gamma_j} a \cdot n \, dS = \sum_{j=0}^{N} F_j = 0$$

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