Extension of holomorphic functions defined on singular complex hypersurfaces with growth estimates

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Abstract. Let $D$ be a strictly convex domain and $X$ be a singular complex hypersurface in $\mathbb{C}^n$ such that $X \cap D \neq \emptyset$ and $X \cap bD$ is transverse. We first give necessary conditions for a function holomorphic on $D \cap X$ to admit a holomorphic extension belonging to $L^q(D)$, with $q \in [1, +\infty]$. When $n = 2$ and $q < +\infty$, we then prove that this condition is also sufficient. When $q = +\infty$ we prove that this condition implies the existence of a $BMO$-holomorphic extension. In both cases, the extensions are given by mean of integral representation formulas and new residue currents.

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1. Introduction

In the last few years, many classical problems in complex analysis have been investigated in the framework of singular spaces; for example the $\bar{\partial}$-Neumann operator has been studied in [34] by Ruppenthal, the Cauchy-Riemann equation in [6,17,21,32,33] by Andersson, Samuelsson, Diederich, Fornæss, Vassiliadou, Ruppenthal, ideals of holomorphic functions on analytic spaces in [5] by Andersson, Samuelsson and Sznajdman, problems of extensions and restrictions of holomorphic functions on analytic spaces in [18,20] by Diederich, Mazzilli and Duquenoy.

In this article we will be interested in problems of extension of holomorphic functions defined on a singular complex hypersurface. Let $D$ be a bounded pseudoconvex domain of $\mathbb{C}^n$ with smooth boundary, let $f$ be a holomorphic function in a neighbourhood of $D$ and let $X = \{ z : f(z) = 0 \}$ be a singular complex hypersurface such that $D \cap X \neq \emptyset$. The first extension problem that one can consider is the following one: Is it true that a function $g$ which is holomorphic on $D \cap X$ has a holomorphic extension to $D$?

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