Weak normality of families of meromorphic mappings and bubbling in higher dimensions

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Abstract. Our primary goal in this paper is to understand whether the sets of normality of families of meromorphic mappings between general complex manifolds are pseudoconvex or not. It turns out that the answer crucially depends on the type of convergence one is interested in. We examine three natural types of convergence introduced by one of us earlier and prove pseudoconvexity of sets of normality for a large class of target manifolds for the so called weak and gamma convergencies. Furthermore we determine the structure of the exceptional components of the limit of a weakly/gamma but not strongly converging sequence, they turn to be rationally connected. This observation allows to determine effectively when a weakly/gamma converging sequence fails to converge strongly. An application to the Fatou sets of meromorphic self-maps of compact complex surfaces is given.

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1. Introduction

1.1. Convergence of meromorphic mappings

When one works with sequences of meromorphic functions and, more generally, mappings one finds himself bounded to consider several notions of their convergence. Some of these notions were introduced in [10] and [18], we shall recall the essentials below. An important question is: what can be said about the maximal open sets where the given sequence converges? It occurs that pseudoconvexity or not of domains of convergence/normality in the case of meromorphic mappings crucially depends on the type of convergence one is looking for.

Now let us briefly describe the ways one can define what does it mean that a sequence \( \{f_k\} \) of meromorphic mappings between complex manifolds \( U \) and \( X \) converges. We start with the most obvious one. A sequence \( \{f_k\} \) of meromorphic mappings between complex manifolds \( U \) and \( X \) is said to converge strongly to a meromorphic map \( f \) if the graphs \( \Gamma_{f_k} \) converge over compacts in \( U \) to the graph

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