**BV minimizers of the area functional in the Heisenberg group under the bounded slope condition**

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**Abstract.** We consider the area functional for \( t \)-graphs in the sub-Riemannian Heisenberg group and study minimizers of the associated Dirichlet problem. We prove that, under a bounded slope condition on the boundary datum, there exists a unique minimizer and that this minimizer is Lipschitz continuous. We also provide an example showing that, in the first Heisenberg group, Lipschitz regularity is sharp even under the bounded slope condition.

**Mathematics Subject Classification (2010):** 49Q20 (primary); 53C17, 49Q05 (secondary).

1. Introduction

The area functional for the \( t \)-graph of a function \( u \in W^{1,1}(\Omega) \) in the sub-Riemannian Heisenberg group \( \mathbb{H}^n \equiv \mathbb{R}_x^n \times \mathbb{R}_y^n \times \mathbb{R}_t \) is

\[
\mathcal{A}(u) := \int_{\Omega} |\nabla u + X^*| \, d\mathcal{L}^{2n},
\]

where \( \Omega \subset \mathbb{R}_x^n \times \mathbb{R}_y^n \) is an open set and \( X^* \) is the vector field

\[
X^*(x, y) := 2(-y, x),
\]

see [13, 46, 52] for more details. It was shown in [52] that the natural variational setting for this functional is the space \( \text{BV}(\Omega) \) of functions with bounded variation in \( \Omega \); more precisely, it was proved that the relaxed functional of \( \mathcal{A} \) in the \( L^1 \)-topology is

\[
\mathcal{A}(u) := \int_{\Omega} |\nabla u + X^*| \, d\mathcal{L}^{2n} + |D^* u|(\Omega), \quad u \in \text{BV}(\Omega),
\]

where \( |D^* u|(\Omega) \) is the total variation in \( \Omega \) of the singular part of the distributional derivative of \( u \).

Received May 14, 2013; accepted in revised version September 26, 2013.