Pointwise estimates and existence of solutions of porous medium and $p$-Laplace evolution equations with absorption and measure data

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Abstract. Let $\Omega$ be a bounded domain of $\mathbb{R}^N (N \geq 2)$. We obtain a necessary and a sufficient condition, expressed in terms of capacities, for the existence of a solution to the porous medium equation with absorption

$$\begin{cases}
    u_t - \Delta (|u|^{m-1}u) + |u|^{q-1}u = \mu & \text{in } \Omega \times (0, T) \\
    u = 0 & \text{on } \partial \Omega \times (0, T) \\
    u(0) = \sigma
\end{cases}$$

where $\sigma$ and $\mu$ are bounded Radon measures, $q > \max(m, 1)$, and $m > \frac{N-2}{N}$. We also obtain a sufficient condition for the existence of a solution to the $p$-Laplace evolution equation

$$\begin{cases}
    u_t - \Delta_p u + |u|^{q-1}u = \mu & \text{in } \Omega \times (0, T) \\
    u = 0 & \text{on } \partial \Omega \times (0, T) \\
    u(0) = \sigma
\end{cases}$$

where $q > p - 1$ and $p > 2$.

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