Completely Sidon sets in discrete groups

Gilles Pisier

Abstract. A subset of a discrete group $G$ is called completely Sidon if its span in $C^*(G)$ is completely isomorphic to the operator space version of the space $\ell_1$ (i.e., $\ell_1$ equipped with its maximal operator space structure). We recently proved a generalization to this context of Drury’s classical union theorem for Sidon sets: completely Sidon sets are stable under finite unions. We give a different presentation of the proof emphasizing the “interpolation property” analogous to the one Drury discovered. In addition we prove the analogue of the Fatou-Zygmund property: any bounded Hermitian function on a symmetric completely Sidon set $\Lambda \subset G \setminus \{1\}$ extends to a positive definite function on $G$. In the final section, we give a completely isomorphic characterization of the closed span $C_\Lambda$ of a completely Sidon set in $C^*(G)$: the dual (in the operator space sense) of $C_\Lambda$ is exact if and only if $\Lambda$ is completely Sidon. In particular, $\Lambda$ is completely Sidon as soon as $C_\Lambda$ is completely isomorphic (by an arbitrary isomorphism) to $\ell_1(\Lambda)$ equipped with its maximal operator space structure.

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